Electromagnetism

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Coulomb's law , force , field , Potential

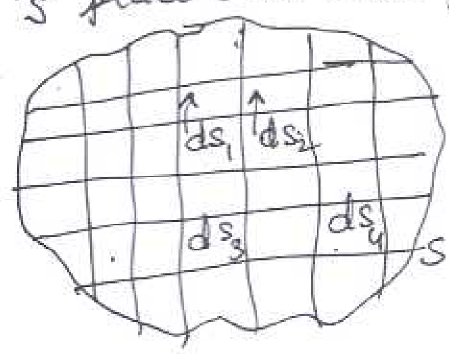
$$\vec{F} = k\frac{q_1 q_2}{r^2} \quad , \quad \vec{E} = \vec{F}/q_2$$

$$E = -\frac{\partial V}{\partial x}$$

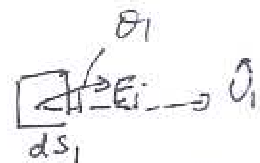
Gauss Law

Normal Induction (flux).

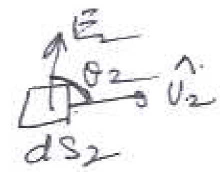
Consider a closed surface 's' placed in some field.



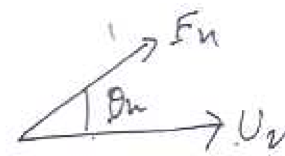
and divide it into smaller parts $ds_1, ds_2 - - -$ ds_n: Let E_1, E_2, be the fields at elements $ds_1, ds_2 - - -$.



Let $\hat{U}_1, \hat{V}_2, U_3$ be the unit normal vectors to the surfaces ds_1, ds_2 - - -



Let $\theta_1, \theta_2, \theta_3$ be the angles between the normals and the direction of field at surfaces ds_1, ds_2, ds_3



At ds_1 Flux is defined as.

$$d\phi_1 = \vec{E_1}\cdot d\vec{s_1} = E_1\, ds_1 \cos\theta_1.$$

ds_2 $$d\phi_2 = \vec{E_2}\cdot d\vec{S_2} = E_2\, dS_2 \cos\theta_2,$$

Total flux. (Total Normal Induction)

$$\phi = E_1 ds_1 \cos\theta_1 + E_2 ds_2 \cos\theta_2 + \dots \dots E_n ds_n \cos\theta_n.$$

$$= \int E.ds.$$

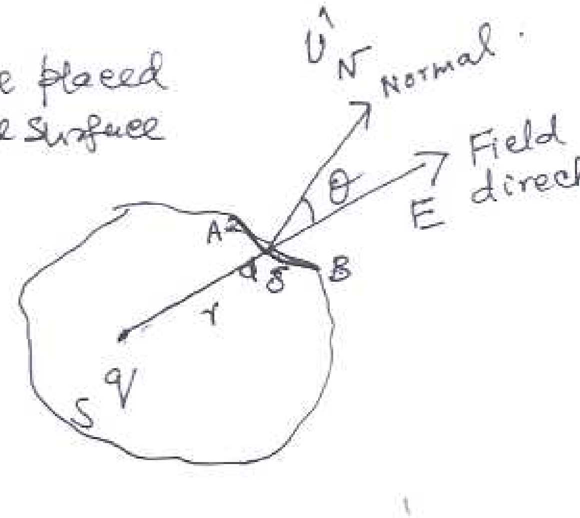
Gauss Law (or Gauss Theorem) states that Total Normal Induction over a closed surface $= q/\epsilon_0$ where q is the total charge placed inside the surface.

$$\boxed{\phi = q/\epsilon_0} \longrightarrow \text{Gauss theorem}$$

$$\downarrow$$

$$\boxed{\int_S E.ds = q/\epsilon_0}$$

Proof of Gauss theorem (i) Charge placed inside surface



Consider a small portion of surface ds and let U_N be the unit normal vector. Let the field be E

Flux passing through surface ds.

$$d\phi = \vec{E}.\vec{ds} = E\,ds\cos\theta$$

The field E at ds due to charge q.

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\therefore d\phi = \frac{q}{4\pi\epsilon_0} \frac{ds\cos\theta}{r^2}.$$

$d\Omega = \frac{ds\cos\theta}{r^2}$ → Solid angle subtended by the surface. ds at ~~q~~ the point where q is placed.

$$d\phi = \frac{q}{4\pi\epsilon_0} d\Omega.$$

Total flux over entire surface S

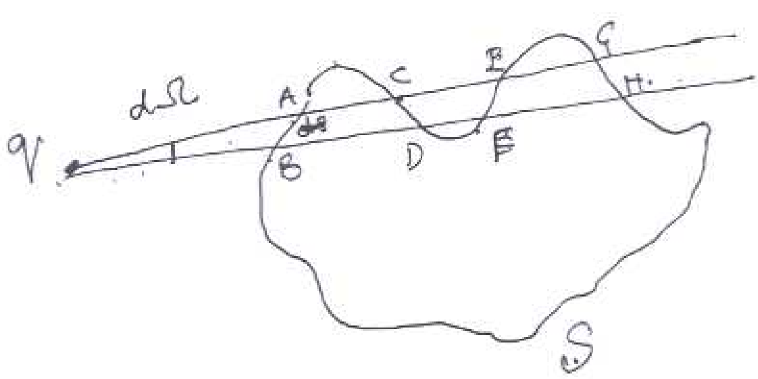
$$\phi = \int d\phi = \frac{q}{4\pi\epsilon_0} \int_S d\Omega.$$

$$\phi = \frac{q}{4\pi\epsilon_0} \times 4\pi =$$

$$\boxed{\phi = q/\epsilon_0}$$

<u>By defintion</u>
outgoing flux is positive
Incoming flux is negative

(ii) <u>Charge is placed outside surface</u>



Consider a small solid angle $d\Omega$ through which flux enters the surface S.

Flux passing through AB $= d\phi_{AB} = -\frac{q}{4\pi\epsilon_0} d\Omega.$

CD $= d\phi_{CD} = +\frac{q}{4\pi\epsilon_0} d\Omega$

EF $= d\phi_{EF} = -q/4\pi\epsilon_0 \, d\Omega$

GH $= d\phi_{GH} = +q/4\pi\epsilon_0 \, d\Omega.$

Total flux $= 0$

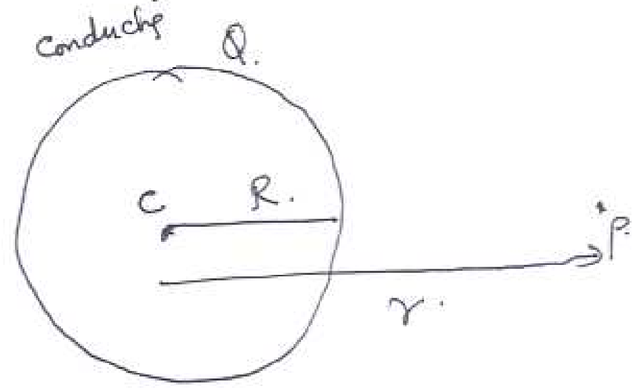
$$\phi = d\phi_{AB} + d\phi_{CD} + \cdots$$

$$= 0.$$

Application of Gauss Law

(1) Conducting sphere of Radius R having total charge Q.

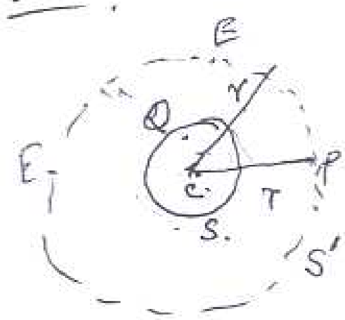
for conducting sphere charge resides on surface.



(1) To find the field at a point P located at a distance 'r' from the centre of sphere.

$CP = r$.

Draw a sphere of radius r, having surface S'. Since S' is symmetrical to the sphere S so field at all points on sphere S' will be same.



$r > R$

Flux passing through S'.

$$\phi = \int_{S'} E \cdot dS = E \int_{S'} dS = E \cdot 4\pi r^2.$$

From Gauss theorem. $\phi = Q/\epsilon_0$.

$$E\, 4\pi r^2 = Q/\epsilon_0$$

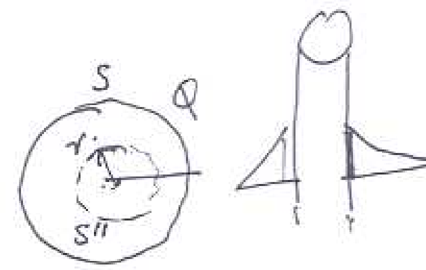
$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}} \rightarrow \text{Field at point } r, \; r > R$$

at $r = R$.

$$\phi = E \cdot 4\pi R^2 = Q/\epsilon_0$$

$$\boxed{E = Q/4\pi\epsilon_0 R^2} \quad \text{at } r = R.$$

(iii) at $\underline{r < R}$



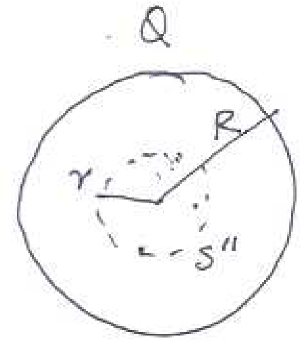
$$\phi = \int_{S''} E \cdot ds$$

$$= E \cdot 4\pi r^2$$

$$= 0 \cdot \text{ from Gauss theorem.}$$

$$\therefore \quad E = 0.$$

Non conducting Solid Sphere



at $r < R$

Define a volume charge density

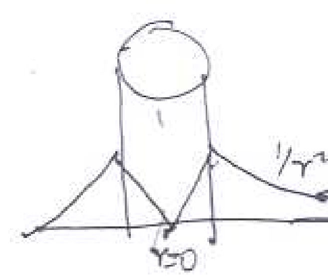
$$\rho = Q / \tfrac{4}{3}\pi R^3.$$

Charge contained in S'' $\quad Q'' = \rho \times \frac{4}{3}\pi r^3$.

Flux passing through S''

$$= \int_{S''} E \cdot ds = E \cdot 4\pi r^2$$

From Gauss theorem $\quad \phi = \dfrac{Q''}{\epsilon_0}$



$$E\, 4\pi r^2 = \frac{\rho \times \frac{4}{3}\pi r^3}{\epsilon_0}.$$

$$E = \frac{\rho \times r}{3\epsilon_0} = \frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{r}{3\epsilon_0} = \frac{Q r}{4\pi R^3 \epsilon_0}$$

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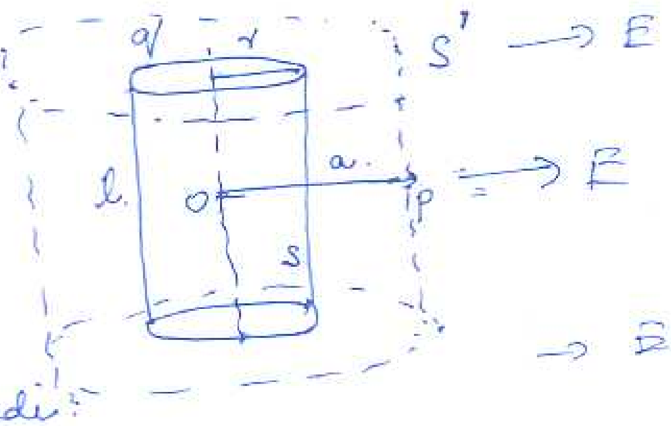
Application of Gauss Theorem

$$\int E.ds = q/\epsilon_0.$$

Cylindrical distributions

(1). Conducting cylinder



Let l be the length and r the radius of cylinder and charge placed on it be q.

(a) To find field at a point distance a. from the axis of cylinder:

$a > r$

By symmetry the field E will be in the direction of OP.

Draw a cylinder with radius a (OP)

Flux coming out of cylinder will be from

(i) Top surface ds_T

(ii) Bottom $\rightarrow$ ds_B

(iii) Curved surface. ds_l.

Flux $d\phi = E.ds = E\,ds\cos\theta$

$\theta \rightarrow$ angle between the normal to the surface and the direction of the field.

Contribution from top surface to flux $d\phi_T = E\,ds_T\cos 90$
$= 0$

" " bottom surface " $d\phi_B = E\,ds_B\cos 90$
$= 0$

Contribution from curved curved surface to flux

$$d\phi_l = E\, ds_l \cos 0^\circ$$

$$= E\, 2\pi a l$$

$\phi \rightarrow$ Total outgoing flux $= E \cdot 2\pi a l + 0 + 0$

According to Gauss theorem.

$$\phi = q/\epsilon_0$$

$$E \cdot 2\pi a l = q/\epsilon_0$$

$$\boxed{E = \frac{q}{2\pi a l \epsilon_0}}$$ for points $a > r$

(ii) if $a = r$

$$E = \cancel{q/2\pi a l \epsilon_0}$$

$$= q/2\pi r l \epsilon_0$$

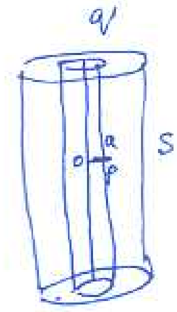
(iii) if $a < r$.

$$E = 0$$

because ~~charge~~ for conducting cylinder the charge resides on the surface and therefore it is external to the points inside the cylinder. According to Gauss theorem.

$$\phi = 0 \quad , \quad E = 0$$

Nonconducting cylinder



Charge q is distributed uniformly over the cylinder.

we define a vol. charge density ρ

$$\rho = q / \pi r^2 l.$$

If the point P is inside the cylinder draw a cylinder of radius a along the point P.

Charge contained inside cylinder of radius a.

$$q' = \rho \times \pi a^2 l.$$

outgoing Flux through the cylinder of radius a.

$$\phi = \int E \cdot ds$$

$$= \cancel{E ds_b + E d}$$

$$= E ds_b \cos 90 + E ds_T \cos 90$$

$$+ E ds_l \cos 0.$$

$$= E ds_l + 0 + 0.$$

$$\phi = E . 2\pi a l.$$

According to Gauss theorem. $\phi = q' / \epsilon_0$

$$E\, 2\pi a l = \frac{\rho \times \pi a^2 l}{\epsilon_0}$$

$$E = \frac{\rho a}{2\epsilon_0}$$

$$= \frac{q}{\pi r^2 l} \cdot \frac{a}{2\epsilon_0}$$

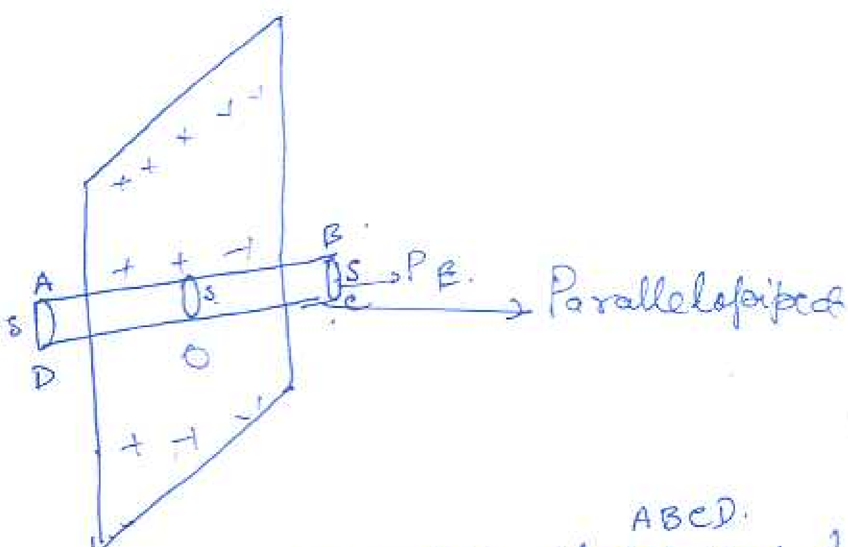
$$\boxed{E = \frac{q a}{2\pi r^2 l\, \epsilon_0}} \rightarrow \underline{\underline{a < r}}$$

Sheet of Charge

(Infinitely long sheet of charge).

Define a surface density of charge

$$\sigma = \frac{\text{Charge}}{\text{Area}}$$



ABCD.

Consider a small surface (Parallelopiped) passing through this sheet.

out going flux from the parallelopiped.

(1) Contribution from AB
BC.
CD
DA

By symmetry the field will along OP.

Contribution from AB → 0 because angle between AB and field direction E is 90°.

" " CD → 0 "

contribution from BC → $ES\cos 0 = ES$

" DA → $ES\cos 0 = ES$

Total outgoing flux.

$$\phi = ES + ES = \underline{\underline{2ES}}.$$

According to Gauss theorem.

$$\phi = q/\epsilon_0.$$

$$2ES = q/\epsilon_0$$

$$= \sigma S/\epsilon_0$$

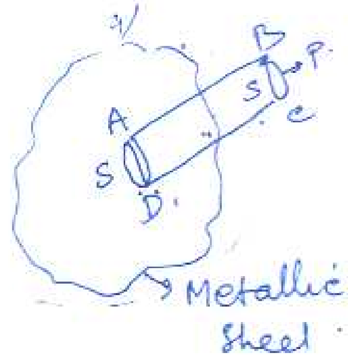
$$\boxed{E = \sigma/2\epsilon_0}$$

This is independent of distance of the point from the charge.

This is because if we are close to the sheet we see less charge (in a small cone from our eye) and as we move away the cone increases and we start seeing more charge and any loss in field due to distance is compensated by the field due to the seeing of more charge.

Metallic sheet of charge

$\sigma \rightarrow$ surface charge density



Flux came out of the parallelopiped.

$\phi_{AB} \longrightarrow 0$ — angle is 90 between AB and field direction

$\phi_{BC} \longrightarrow ES\cos 0$
$= ES.$

$\phi_{CD} \rightarrow 0$

$\phi_{DA} \rightarrow 0 \rightarrow$ because charge lies outside the surface.

Total flux

$$\phi = ES$$

$$ES = q/\epsilon_0$$

$$= \sigma S/\epsilon_0$$

$$\boxed{E = \sigma/\epsilon_0}$$

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Poisson's Equation and Laplace Equation

1. Electric Flux Density.

$$D = \epsilon \underline{E}$$

$\epsilon = \epsilon_0 \rightarrow$ free space

$\epsilon \rightarrow$ permittivity of the medium.

From Gauss theorem,

$$\phi = \int_S E \cdot ds = q/\epsilon_0$$

$$\int_S \vec{D} \cdot \vec{ds} = \underline{q} \rightarrow \text{Gauss theorem}$$

vol. charge density $\rho = q/v$

$$q = \rho \underline{v} = \int_v \rho \, d\underline{v}$$

$$\int_S \vec{D} \cdot d\vec{s} = \int_v \rho \, d\underline{v} \quad \text{—— (1)}$$

From Gauss Divergence theorem

$$\int_S \vec{D} \cdot d\vec{s} = \int_v (\vec{\nabla} \cdot \vec{D}) \, d\underline{v} \rightarrow (2)$$

Comparing eq (1) and (2)

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \epsilon_0 \vec{E} = \rho$$

$$\vec{E} = - \vec{\nabla} V$$

$$\vec{\nabla} \cdot \vec{\nabla} V = - \rho/\epsilon_0$$

$$\boxed{\nabla^2 V = - \rho/\epsilon_0} \rightarrow \text{Poisson's Equation.}$$

Gradient operator

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\boxed{\nabla^2 V = -\rho/\epsilon_0} \longrightarrow \text{Poisson Equation}$$

$\nabla^2 \rightarrow$ Laplacian operator.

$\nabla^2 \rightarrow$ Cartesian Coordinates $= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\nabla^2 \rightarrow$ Spherical polar coordinates (r, θ, ϕ)

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Laplace Equation

$$\boxed{\nabla^2 V = 0}$$

If Vol. charge density $\rho = 0$

Laplace Equation.

Surface charge density, or line charge density ~~st~~ may still exist.

$$\nabla^2 V = 0.$$

Suppose we are considering one dimensional system.

$$\nabla^2 = \frac{\partial^2}{\partial x^2}$$

$$\frac{\partial^2 V}{\partial x^2} = 0,$$

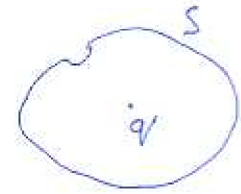
$$\frac{\partial V}{\partial x} = \text{Constant} = c$$

$$\boxed{V = cx + d.}$$

Solution of one dimensional Laplace equation.

Laws of Electromagnetism

(i) Gauss Law of Electricity

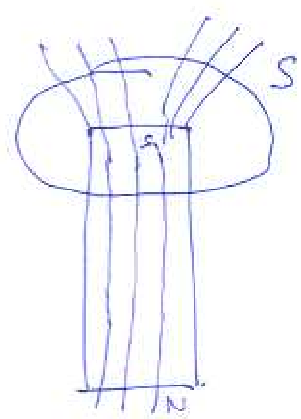


$$\text{Electric flux} = \int_S \vec{E}\cdot d\vec{s} = q/\epsilon_0$$

(ii) Gauss Law of Magnetism.

$$\text{Magnetic flux } \phi = \int_S \vec{B}\cdot d\vec{s} = 0$$

Consider a surface S around one of the poles of a magnet.



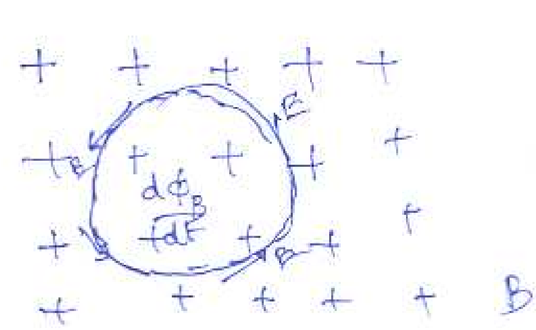
Magnetic lines of force travel inside the magnet from one pole and emerges out from the other pole. So the flux enters through the surface and the same flux emerges out of the surface. Therefore the net flux over the surface S = 0.

(iii) Faraday's Law

Changing magnetic field gives rise to electric field.

Induced E.m.f in the loop due to changing magnetic flux

$$\mathcal{E} = -\frac{\partial \phi_B}{\partial t}$$



(Rate of change of flux is the E.m.f induced in circuit)

If $q_0 \to$ is the charge taken round the loop

$$F = q_0 E$$

work done $\quad F \cdot dx$
in taking the charge around the loop

$$W = \underline{\underline{q_0 E \, 2\pi r}}.$$

$\downarrow$

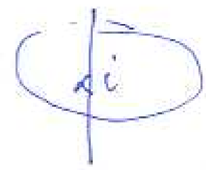
$$\cancel{q_0} \mathcal{E} = \cancel{q_0} E \, 2\pi r$$

$$-\frac{\partial \phi_B}{\partial t} = \int_\Gamma \vec{E} \cdot d\vec{l}$$

$$\boxed{\int \vec{E} \cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t}.}$$

(IV) Ampere's Law

Magnetic field is produced around a current carrying wire :



$$B \propto i/r$$

$$B = \frac{\mu_0}{2\pi} \frac{i}{r} \qquad \to \mu_0 \to \text{permeability of the medium.}$$

$$B \cdot 2\pi r = \mu_0 i$$

$$\boxed{\int \vec{B} \cdot d\vec{l} = \mu_0 i}$$

$$\int E \cdot dS = q/\epsilon_0 \quad \text{—(1)}$$

$$\int B \cdot dS = 0 \quad \rightarrow (2)$$

$$\int \vec{E} \cdot d\vec{l} = -\partial \phi_B / \partial t \quad \rightarrow (3)$$

$$\int \vec{B} \cdot d\vec{l} = \mu_0 i \quad \text{—(4)}$$

Asymmetry in the above laws

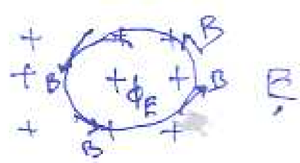
I. In eq (1) and (2), the r.H.S of (2) contains no value, because of the nonexistence of magnetic monopoles

II. In eq (4) rHS contains $\mu_0 i \rightarrow \mu_0 \frac{dq}{dt}$ but there is no such term in eq (3). ∴ corresponding to $\mu_0 i$. This asymmetry is also because of nonexistence of magnetic monopoles.

III. In eq. (3) changing magnetic flux gives rise to electric field. There is no such corresponding term relating to changing electric ~~field~~ flux producing magnetic field in eq(4). This led Maxwell to introduce a new term corresponding to changing electric flux in eq(4)

The corresponding term.

$$\int \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$



$$\boxed{\int B \cdot dl = \mu_0 i + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}}$$

$$= \mu_0 (i + i_d)$$

$i_d \rightarrow$ displacement current. $i_d = \epsilon_0 \frac{d\phi_E}{dt}$

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Maxwell's equations

E & B (Integral form)

$$\int E \cdot ds = q/\epsilon_0 \quad -(1)$$

$$\int B \cdot ds = 0 \quad -(2)$$

$$\int E \cdot dl = -\partial \phi_B / \partial t \quad (3)$$

$$\int B \cdot dl = \mu_0 (i + \epsilon_0 \partial \phi_E / \partial t) \quad -(4)$$

Differential form

use (1) equation and apply Gauss divergence theorem

$$\int \vec{E} \cdot d\vec{s} = \int_v \vec{\nabla} \cdot \vec{E} \, dv.$$

$$\int_v (\nabla \cdot E) \, dv = \frac{q}{\epsilon_0} = \int_v \frac{\rho \, dv}{\epsilon_0}$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0} \quad \text{——} \quad (1)$$

(2) $$\int_s B \cdot ds = \int_v \vec{\nabla} \cdot \vec{B} \, dv = 0$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0} \quad \text{——} \quad (2)$$

(3) Use stoke's theorem.

$$\int_L \vec{E} \cdot d\vec{l} = \int_s (\vec{\nabla} \times \vec{E}) \cdot d\vec{s}.$$

using this in eq (3)

$$\phi_B = BA$$

$$\int_s (\vec{\nabla} \times \vec{E}) \cdot ds = -\frac{\partial \phi_B}{\partial t} = -\frac{\partial}{\partial t}(BA) \qquad A = \int ds$$

$$= -\frac{\partial B}{\partial t} A = -\int \frac{dB}{dt} \cdot ds$$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}} \longrightarrow (3)$$

(4) use Stoke's theorem.

$$\int B \cdot dl = \int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{s}$$

current density

$$J = \frac{i}{A}$$

$$\int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{s} = \mu_0 \left[\underset{i}{JA} + \epsilon_0 \frac{\partial}{\partial t} (\phi_E) \right]$$

$$= \mu_0 \left[JA + \epsilon_0 \frac{\partial}{\partial t} (EA) \right]$$

$$= \int \mu_0 \left[J + \epsilon_0 \frac{\partial E}{\partial t} \right] \cdot dS \qquad A = \int ds$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left[J + \epsilon_0 \partial E / \partial t \right] \quad \text{--- (4)}$$

$D = \epsilon_0 E$ $\rightarrow$ $D \rightarrow$ Electric flux density.

$$= \mu_0 \left[J + \partial D / \partial t \right]$$

<u>Maxwell's equations in differential form</u>

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0 \quad \text{--- (1)}$$
$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{--- (2)}$$
$$\vec{\nabla} \times \vec{E} = - \partial B / \partial t \quad \text{--- (3)}$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \left[J + \epsilon_0 \partial E / \partial t \right] \quad \text{--- (4)}$$

Suppose we have free space.

(No free charges and no current.)

Free space: $\rho = 0$ $\quad J = 0$ $\quad q = 0, i = 0$

Maxwell's equations in free space

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \text{--- (1)}$$
$$\vec{\nabla} \cdot \vec{B} = 0 \quad (2)$$
$$\vec{\nabla} \times \vec{E} = - \partial B / \partial t \quad \text{--- (3)}$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \partial E / \partial t. \quad (4)$$

use eq (3). and apply $\vec{\nabla}\times$ to it

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\left(\vec{\nabla}\times\frac{\partial B}{\partial t}\right)$$

$$\underbrace{\vec{\nabla}(\vec{\nabla}\cdot\vec{E})}_{0} - \nabla^2 E = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

using eq (4)

$$0 - \nabla^2 E = -\frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

$y = a\sin(kx-\omega t)$

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

The Electric field vector E follows a wave equation which has a velocity

$$\frac{1}{v^2} = \mu_0 \epsilon_0$$

$$v^2 = 1/\mu_0\epsilon_0$$

The Electric vector vibrates with a wavelike motion and progresses with a velocity v

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Put the values of μ_0 and ϵ_0

$$\mu_0 = 4\pi\times10^{-7}$$

$$\frac{1}{4\pi\epsilon_0} = 9\times10^{9}$$

$$v = 3\times10^{8}\ m/s.$$

which is same as the velocity of light.

Light is electromagnetic in nature with electric vector E and magnetic vector B oscillating as wave and propagating with velocity of light.

For magnetic field ; apply $\nabla \times$ to eq (4).

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 \epsilon_0 \left(\vec{\nabla} \times \frac{\partial E}{\partial t} \right).$$

$$\nabla (\vec{\nabla} \cdot \vec{B}) - \nabla^2 B = \mu_0 \epsilon_0 \left(\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) \right)$$

use eq (3)

$$0 - \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\partial B / \partial t \right).$$

$$\boxed{\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}}$$

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

Both E and B follow wave equation.

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

$$\boxed{v^2 = \frac{1}{\mu_0 \epsilon_0}}$$

Solution of wave equation

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}, \quad \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

This has harmonic solutions

$$E = E_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

$$B = B_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

(1) $\vec{\nabla} \cdot \vec{E} = 0$ $\qquad -ik \cdot E_0 e^{i(\omega t - \vec{k} \cdot \vec{r})} = 0$

$$= -i\vec{k} \cdot \vec{E} = 0$$

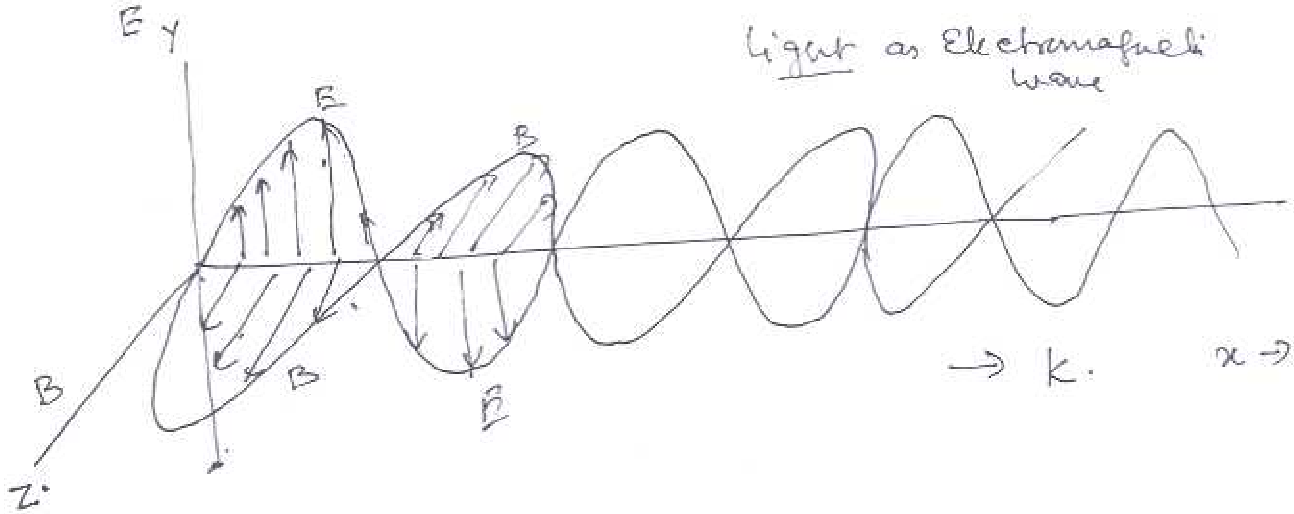
k and E are $\perp^r$ to each other.

(2) $\vec{\nabla} \cdot \vec{B} = 0$ $\qquad -i\vec{k} \cdot \vec{B} = 0$

k and B are $\perp^r$ to each other.

k, E and B → are $\perp^r$ to each other.

$k \rightarrow$ propagation vector



What is the relation between E and B

(3) $\vec{\nabla} \times \vec{E} = -\partial B/\partial t$

$\downarrow$

$-i\vec{k} \times \vec{E} = -i\omega B$

$\vec{k} \times \vec{E} = \omega B.$

$E \rightarrow E_y$

$B \rightarrow B_z$

$k \rightarrow x.$

$k E_y = \omega B_z.$

$E_y = \frac{\omega}{k} B_z.$

$E_y = c B_z$

$E_y = cB_z.$

or $B_z = E_y/c.$

$k = \frac{2\pi}{\lambda}$

$\omega = 2\pi\nu$

$\nu = c/\lambda$

$E_y = E_0 e^{i(\omega t - kx)}$

$B_z = B_0 e^{i(\omega t - kx)}$

$$\boxed{B_0 = E_0/c}$$

Both B and E are in phase.